

# Energy Balances

“Energy” used to be a term that everybody took for granted. Now, to a layman, energy has been added to the list of the basic necessities of life. Increasing energy costs have forced people to recognize and appreciate the value of energy more than ever before. Energy conservation is being stressed not only in industrial operations but also in almost all aspects of an individual’s daily activities.

Energy is not static; it is always in a stage of flux. Even under steady-state conditions, an object absorbs energy from its surroundings and at the same time emits energy to its surroundings at the same rate. When there is an imbalance between the energy absorbed and emitted, the steady state is altered, molecular energy of some parts of the system may increase, new compounds may be formed, or work may be performed. Energy balance calculations can be used to account for the various forms of energy involved in a system.

Energy audits are essential in identifying effectiveness of energy conservation measures and in identifying areas where energy conservation can be done. The technique is also useful in the design of processing systems involving heating or cooling to insure that fluids used for heat exchange are adequately provided and that the equipment is sized adequately to achieve the processing objective at the desired capacity. When energy exchange involves a change in mass due to evaporation or condensation, energy balances can be used during formulation such that after processing, the product will have the desired composition.

## 5.1 GENERAL PRINCIPLES

An energy balance around a system is based on the first law of thermodynamics: the law of conservation of energy. Mechanical (work), electrical, and thermal energy can all be reduced to the same units. Mechanical input into a system to overcome friction, electrical energy, or electromagnetic energy such as microwaves will be manifested by an increase in the heat content of the system. Defining the surroundings of the system on which to make an energy balance is done similar to material balance calculations in Chapter 3. The basic energy balance equation is

$$\text{Energy in} = \text{Energy out} + \text{Accumulation} \quad (5.1)$$

If the system is in a steady state, the accumulation term is zero, whereas an unsteady-state system will have the accumulation term as a differential expression. When Equation (5.1) is used, all energy terms known to change within the system must be accounted for. The heat contents are expressed as

enthalpy based on increase in enthalpy from a set reference temperature. Mechanical, electrical, or electromagnetic inputs must all be accounted for if their effects on the total heat content are significant. If the system involves only an exchange of energy between two components, the energy balance will be:

$$\text{Energy gain by component 1} = \text{Energy loss by component 2} \quad (5.2)$$

Either Equation (5.1) or (5.2) will give similar results if Equation (5.2) is applicable. Equation (5.1) is a general form of the energy balance equation.

## 5.2 ENERGY TERMS

The unit of energy in SI is the Joule. Conversion factors in Appendix Table A.1 can be used to convert mechanical and electrical energy units to the SI equivalent. Microwave and radiant energy absorbed by a material are usually expressed as a rate of energy flow, energy/time. Energy from ionizing radiation is expressed as an absorbed dose, the Gray (Gy), which has the base units J/kg. Another accepted form for reporting absorbed ionizing radiation is the rad,  $100 \text{ rd} = 1 \text{ Gy}$ .

### 5.2.1 Heat

Sensible heat is defined as the energy transferred between two bodies at different temperatures, or the energy present in a body by virtue of its temperature. Latent heat is the energy associated with phase transitions, heat of fusion, from solid to liquid, and heat of vaporization, from liquid to vapor.

### 5.2.2 Heat Content, Enthalpy

Enthalpy, as defined in Chapter 4, is an intrinsic property, the absolute value of which cannot be measured directly. However, if a reference state is chosen for all components that enter and leave a system such that at this state the enthalpy is considered to be zero, then the change in enthalpy from the reference state to the current state of a component can be considered as the value of the absolute enthalpy for the system under consideration. The reference temperature ( $T_{\text{ref}}$ ) for determining the enthalpy of water in the steam tables (Appendix Table A.3 and A.4) is  $32.018^\circ\text{F}$  or  $0.01^\circ\text{C}$ . The enthalpy of any component of a system that would be equivalent to the enthalpy of water obtained from the steam tables, at any temperature  $T$  is given by:

$$H = C_p(T - T_{\text{ref}}) \quad (5.3)$$

$C_p$  in Equation (5.3) is the specific heat at constant pressure, previously defined in Chapter 4.

### 5.2.3 Specific Heat of Solids and Liquids

The specific heat ( $C_p$ ) is the amount of heat that accompanies a unit change in temperature for a unit mass. The specific heat, which varies with temperature, is more variable for gases compared with liquids or solids. Most solids and liquids have a constant specific heat over a fairly wide temperature

range. The enthalpy change of a material with mass  $m$  is:

$$q = m \int_{T_1}^{T_2} C_p dT \quad (5.4)$$

Handbook tables give specific heats averaged over a range of temperature. When average specific heats are given, Equation (5.4) becomes:

$$q = m C_{\text{avg}} (T_2 - T_1) \quad (5.5)$$

For solids and liquids, Equations (5.3) and (5.5) are valid over the range of temperatures encountered in food processing systems.

Table 5.1 shows the average specific heats of various solids and liquids.

For fat-free fruits and vegetables, purees and concentrates of plant origin, Siebel (1918) observed that the specific heat varies with moisture content and that the specific heat can be determined as the weighted mean of the specific heat of water and the specific heat of the solids.

For a fat-free plant material with a mass fraction of water  $M$ , the specific heat of water above freezing is 1 BTU/(lb  $\cong$  °F) or 4186.8 J/(kg  $\cong$  K), and that of non-fat solids is 0.2 BTU/(lb  $\cong$  °F) or 837.36 J/(kg  $\cong$  K). Because the mass fraction of non-fat solids is  $(1 - M)$ , the weighted average specific heat for unit mass of material above freezing is

$$C'_{\text{avg}} = 1(M) + 0.2(1 - M) = (1 - 0.2)M + 0.2 = 0.8 M + 0.2 \quad \text{in BTU/(lb } ^\circ\text{F)} \quad (5.6)$$

In SI:

$$C_{\text{avg}} = 3349 M + 837.36 \quad \text{in J/(kg K)} \quad (5.7)$$

Equations (5.6) and (5.7) are different forms of Seibel's equation, which has been used by Ashrae (1965) in tabulated values for specific heat of fruits and vegetables.

When fat is present, the specific heat above freezing may be estimated from the mass fraction fat (F), mass fraction solids non-fat (SNF), and mass fraction moisture (M), as follows:

$$C_{\text{avg}} = 0.4 F + 0.2 \text{ SNF} + M \quad \text{in BTU/(lb } ^\circ\text{F)} \quad (5.8)$$

$$C'_{\text{avg}} = 1674.72 F + 837.36 \text{ SNF} + 4186.8 M \quad \text{in J/(kg K)} \quad (5.9)$$

Below freezing, it is not suitable to use specific heats for the whole mixture because the amount of frozen and unfrozen water vary at different temperatures. It will be necessary to consider the latent heat of fusion of water, and sensible heats of liquid water and ice should be evaluated separately. Refer to the section "Enthalpy Changes in Foods During Freezing" later in this chapter.

Equations (5.8) and (5.9) are general and can be used instead of Equations (5.6) and (5.7).

**Example 5.1.** Calculate the specific heat of beef roast containing 15% protein, 20% fat, and 65% water.

**Solution:**

$$C'_{\text{avg}} = 0.15(0.2) + 0.20(0.4) + 0.65(1) = 0.76 \text{ BTU/(lb } ^\circ\text{F)}$$

$$C_{\text{avg}} = 0.15(837.36) + 0.2(1674.72) + 0.65(4186.8) = 3182 \text{ J/(kg K)}$$

**Table 5.1** Specific Heat of Food Products.

<i>Product</i>	<i>% H<sub>2</sub>O</i>	<i>C<sub>pm</sub></i>
Dairy products		
Butter	14	2050
Cream, sour	65	2930
Milk, skim	91	4000
Fresh meat, fish, poultry, and eggs		
Codfish	80	3520
Chicken	74	3310
Egg white	87	3850
Egg yolk	48	2810
Pork	60	2850
Fresh fruits, vegetables, and juices		
Apples	75	3370
Apple juice	88	3850
Apple sauce	—	3730
Beans, fresh	90	3935
Cabbage, white	91	3890
Carrots	88	3890
Corn, sweet, kernels	—	3320
Cucumber	97	4103
Mango	93	3770
Orange juice, fresh	87	3890
Plums, fresh	76.5	3500
Spinach	87	3800
Strawberries	91	3805
Other products		
Bread, white	44	22720
Bread, whole wheat	48.5	2850
Flour	13	1800

Source: Adapted from Polley, S. L. Snyder, O. P., and Kotnour, P. A. Compilation of thermal properties of foods. *Food Technol.* 36(1):76, 1980.

**Example 5.2.** Calculate the specific heat of orange juice concentrate having a solids content of 45%.

**Solution:**

Using Siebel's equation: Calculating a weighted average specific heat:

$$C'_{\text{avg}} = 0.2(0.45) + 1(0.55) = 0.64 \text{ BTU}/(\text{lb } ^\circ\text{F})$$

$$C_{\text{avg}} = 837.36(0.45) + 4186.8(0.55) = 2679 \text{ J}/(\text{kg K})$$

**Example 5.3.** Calculate the heat required to raise the temperature of a 4.535 kg (10 lb) roast containing 15% protein, 20% fat, and 65% water from 4.44°C (40°F) to 65.55°C (150°F). Express this energy in (a) BTU, (b) joules, and (c) watt-hour.

**Solution:**

$$C'_{\text{avg}} \text{ from Example 5.1} = 0.76 \text{ BTU}/(\text{lb} \cdot ^\circ\text{F})$$

$$\begin{aligned} \text{(a) } q &= mC_{\text{avg}}(T_2 - T_1) \\ &= 10 \text{ lb} \frac{0.76 \text{ BTU}}{\text{lb} \cdot ^\circ\text{F}} (150 - 40)^\circ\text{F} = 836 \text{ BTU} \end{aligned}$$

$$\text{(b) Use } C_{\text{avg}} \text{ from Example 5.1} = 3182 \text{ J}/(\text{kg} \cdot \text{K})$$

$$q = 4.535 \text{ kg}[3182 \text{ J}/(\text{kg} \cong \text{K})] (65.55 - 4.44) \text{ K} = 0.882 \text{ MJ}$$

$$\text{(c) } q = 0.882 \text{ MJ} \cdot \frac{10^6}{\text{MJ}} \cdot \frac{1 \text{ Ws}}{\text{J}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = 245 \text{ W} \cdot \text{h}$$

The specific heat calculated by Siebel's equation is used by the American Society for Heating, Refrigerating, and Air Conditioning Engineers in one of the most comprehensive tabulated values for specific heat of foodstuffs. However, it is overly simplified, and the assumption that all types of non-fat solids have the same specific heat may not always be correct. Furthermore, Siebel's equation for specific heat below the freezing point assumes that all the water is frozen, and this is most inaccurate.

Specific heats of solids and liquids may also be estimated using correlations obtained from Choi and Okos (1987). The procedure is quite unwieldy to do by hand, however, the data may be entered into a spreadsheet to facilitate repetitive calculations. The specific heats, in J/(kg · K), as a function of T (°C) for various components of foods are as follows:

$$\text{Protein: } C_{\text{pp}} = 2008.2 + 1208.9 \times 10^{-3} T - 1312.9 \times 10^{-6} T^2$$

$$\text{Fat: } C_{\text{pf}} = 1984.2 + 1473.3 \times 10^{-3} T - 4800.8 \times 10^{-6} T^2$$

$$\text{Carbohydrate: } C_{\text{pc}} = 1548.8 + 1962.5 \times 10^{-3} T - 5939.9 \times 10^{-6} T^2$$

$$\text{Fiber: } C_{\text{pfi}} = 1845.9 + 1930.6 \times 10^{-3} T - 4650.9 \times 10^{-6} T^2$$

$$\text{Ash: } C_{\text{pa}} = 1092.6 + 1889.6 \times 10^{-3} T - 3681.7 \times 10^{-6} T^2$$

$$\text{Water above freezing: } C_{\text{waf}} = 4176.2 - 9.0864 \times 10^{-5} T + 5473.1 \times 10^{-6} T^2$$

The specific heat of the mixture above freezing is

$$C_{\text{avg}} = P(C_{\text{pp}}) + F(C_{\text{pf}}) + C(C_{\text{pc}}) + \text{Fi}(C_{\text{pfi}}) + A(C_{\text{pa}}) + M(C_{\text{waf}}) \quad (5.10)$$

where P, F, Fi, A, C and M, represent the mass fraction of protein, fat, fiber, ash, carbohydrate and moisture, respectively.

A spreadsheet for calculating specific heat at one temperature using the data given in Example 5.4 is shown in Fig. 5.1.

**Example 5.4.** Calculate the specific heat of a formulated food product that contains 15% protein, 20% starch, 1% fiber, 0.5% ash, 20% fat, and 43.5% water at 25°C.

**Solution:**

The calculated values, in J/(kg  $\cong$  K), are  $C_{\text{pp}} = 2037.6$ ;  $C_{\text{pf}} = 2018.0$ ;  $C_{\text{pc}} = 1594.1$ ;  $C_{\text{pfi}} = 1891.3$ ;  $C_{\text{pa}} = 1137.5$ ; and  $C_{\text{waf}} = 4179.6$  Substituting in Equation (5.14)

$$\begin{aligned} C_{\text{pavg}} &= 0.15(2037.6) + 0.2(1594.1) + 0.01(1891.3) + 0.005(1137.5) \\ &\quad + 0.2(2018) + 0.435(4179.6) = 2870.8 \text{ J}/(\text{kg} \cong \text{K}) \end{aligned}$$

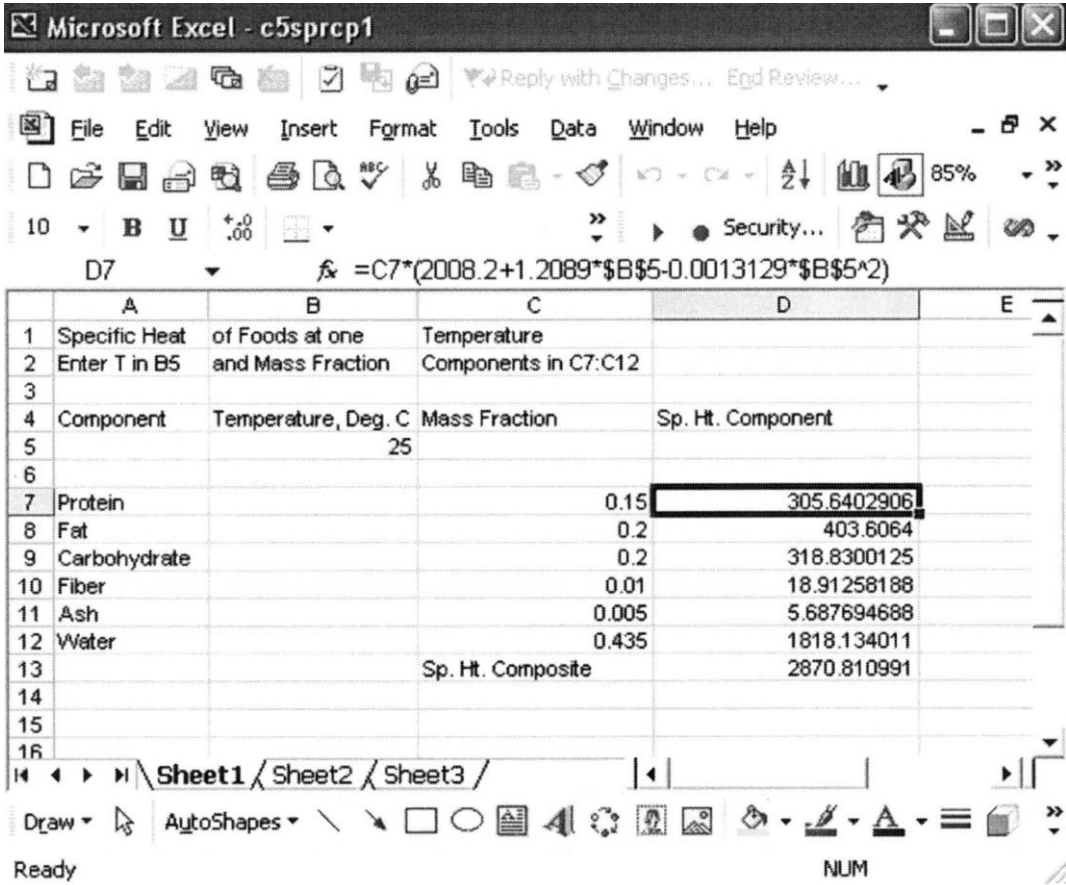


Figure 5.1 Spreadsheet for calculating specific heat using data in Example 5.4.

By comparison, using Siebel's equation:

$$C_p = 1674.72(0.2) + 837.36(0.15 + 0.01 + 0.005 + 0.2) + 4186.8(0.435) = 2462 \text{ J/(kg K)}$$

Values for  $C_p$  calculated using Choi and Okos' (1988) correlations are generally higher than those calculated using Siebel's equation at high moisture contents.

Siebel's equations have been found to agree closely with experimental values when  $M > 0.7$  and when no fat is present. Choi and Okos' correlation is more accurate at low moisture contents and for a wider range of product composition because it is based on published literature values for a wide variety of foods. The simplicity of Siebel's equations however appeals to most users particularly when tolerance for error is not too stringent.

For enthalpy change calculations, Choi and Okos' equations for specific heat must be expressed as an average over the range of temperatures under consideration. The mean specific heat,  $C^*$ , over a

temperature range  $T_1$  to  $T_2$ , where  $(T_2 - T_1) = \delta$ ,  $T_2^2 - T_1^2 = \delta^2$  and  $T_2^3 - T_1^3 = \delta^3$  is

$$C^* = \frac{1}{\delta} \int_{T_1}^{T_2} C_p dT$$

Thus, the equations for the mean specific heats of the various components over the temperature range  $\delta$ , become

$$\text{Fiber: } C_{\text{pfi}}^* = [1/\delta][1845.9(\delta) + 0.9653(\delta^2) - 1550 \times 10^{-6}(\delta^3)]$$

$$\text{Ash: } C_{\text{pa}}^* = [1/\delta][1092.6(\delta) + 0.9448(\delta^2) - 1227 \times 10^{-6}(\delta^3)]$$

$$\text{Water: } C_{\text{waf}}^* = [1/\delta][4176.2(\delta) - 4.543 \times 10^{-5}(\delta^2) + 1824 \times 10^{-6}(\delta^3)]$$

$$\text{Protein: } C_{\text{pp}}^* = [1/\delta][2008.2(\delta) + 0.6045(\delta^2) - 437.6 \times 10^{-6}(\delta^3)]$$

$$\text{Fat: } C_{\text{pf}}^* = [1/\delta][1984.2(\delta) + 0.7367(\delta^2) - 1600 \times 10^{-6}(\delta^3)]$$

$$\text{Carbohydrate: } C_{\text{pc}}^* = [1/\delta][1548.8(\delta) + 0.9812(\delta^2) - 1980 \times 10^{-6}(\delta^3)]$$

$$C_{\text{avg}}^* = P(C_{\text{pp}}^*) + F(C_{\text{pf}}^*) + C(C_{\text{pc}}^*) + Fi(C_{\text{pfi}}^*) + A(C_{\text{pa}}^*) + M(C_{\text{waf}}^*) \quad (5.11)$$

**Example 5.5.** Calculate the mean specific heat of the formulated food product in example 4, in the temperature range  $25^\circ\text{C}$  to  $100^\circ\text{C}$ .

**Solution:**

$\delta = 75^\circ\text{C}$ ,  $\delta^2 = 9375^\circ\text{C}^2$ ,  $\delta^3 = 984375^\circ\text{C}^3$ . The mean specific heats of the components in  $\text{J}/(\text{kg} \cong \text{K})$ , are  $C_{\text{pp}}^* = 2078$ ;  $C_{\text{pf}}^* = 2055.3$ ;  $C_{\text{pc}}^* = 1645.4$ ;  $C_{\text{pfi}}^* = 1948.7$ ;  $C_{\text{pa}}^* = 1137.5$ ;  $C_{\text{waf}}^* = 4200$ . The mean specific heat:  $C_{\text{pavg}}^* = 2904 \text{ J}/(\text{kg} \cdot \text{K})$ . The spreadsheet for calculating average specific heat over a temperature range using the data in Example 5.5 is shown in Fig. 5.2.

### 5.3 ENTHALPY CHANGES IN FOODS DURING FREEZING

#### 5.3.1 Correlation Equations Based on Freezing Points of Food Products Unmodified from the Natural State

When considering the heat to be removed during freezing of a food product, a change in phase is involved, and the latent heat of fusion must be considered. Water in a food does not all change into ice at the freezing point. Some unfrozen water exists below the freezing point, and therefore Siebel's equations for specific heat below the freezing point is very inaccurate. The best method for determining the amount of heat that must be removed during freezing, or the heat input for thawing, is by calculating the enthalpy change. One method for calculating enthalpy change below the freezing point (*good only for moisture contents between 73% and 94%*) is the procedure of Chang and Tao (1981). In this correlation, it is assumed that all water is frozen at  $227 \text{ K}$  ( $-50^\circ\text{F}$ ).

A reduced temperature ( $T_r$ ) is defined as:

$$T_r = \frac{T - 227.6}{T_f - 227.6} \quad (5.12)$$



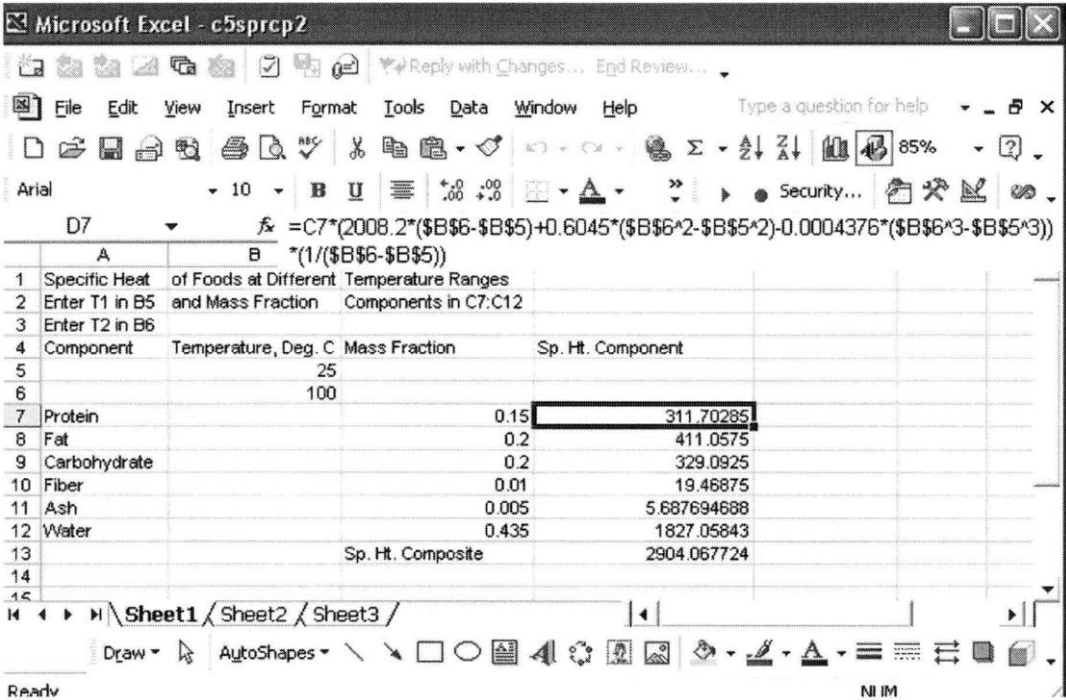


Figure 5.2 Spreadsheet for calculating specific heat as a function of temperature using data in Example 5.5.

where  $T_f$  is the freezing point temperature and  $T$  is the temperature at which the enthalpy is being determined. Two parameters,  $a$  and  $b$ , have been calculated for different products as a function of the mass fraction of moisture in the product,  $M$ . The correlation equations are as follows.

Meats:

$$a = 0.316 - 0.247(M - 0.73) - 0.688(M - 0.73)^5 \quad (5.13)$$

$$b = 22.95 + 54.68(a - 0.28) - 5589.03(a - 0.28)^5 \quad (5.14)$$

Vegetables, fruits, juices:

$$a = 0.362 + 0.0498(M - 0.73) - 3.465(M - 0.73)^2 \quad (5.15)$$

$$b = 27.2 - 129.04(a - 0.23) - 481.46(a - 0.23)^2 \quad (5.16)$$

The freezing point ( $T_f$ ), in K is

$$\text{Meats : } T_f = 271.18 + 1.47 M \quad (5.17)$$

$$\text{Fruits and vegetables: } T_f = 287.56 - 49.19 M + 37.07 M^2 \quad (5.18)$$

$$\text{Juices : } T_f = 120.47 + 327.35 M - 176.49 M^2 \quad (5.19)$$

The enthalpy at the freezing point,  $H_f$ , in J/kg, relative to 227.6 K is

$$H_f = 9792.46 + 405096 M \quad (5.20)$$



The enthalpy at temperature  $T$  relative to 227.6 K is determined by:

$$H = H_f \left[ a T_r + (1 - a) T_r^b \right] \quad (5.21)$$

**Example 5.6.** Calculate the freezing point and the amount of heat that must be removed in order to freeze 1 kg of grape juice containing 25% solids from the freezing point to  $-30^\circ\text{C}$ .

**Solution:**

$$Y = 0.75.$$

Using Equation (5.19) for juices:

$$T_f = 120.47 + 327.35(0.75) - 176.49(0.75)^2 = 266.7 \text{ K}$$

Using Equation (5.20):

$$H_f = 9792.46 + 405,096(0.75) = 313,614 \text{ J}$$

Using Equation (5.15):

$$a = 0.362 + 0.0498(0.02) - 3.465(0.02)^2 = 0.3616$$

Using Equation (5.16):

$$b = 27.2 - 129.04(0.1316) - 481.46(0.1316)^2 = 1.879$$

$$T_r = (-30 + 273 - 227.6)/(266.7 - 227.6) = 0.394$$

Using Equation (5.21):

$$H = 313,614 \left[ (0.3616)0.394 + (1 - 0.3616)(0.394)^{1.879} \right] = 79,457 \text{ J/kg}$$

The enthalpy change from  $T_f$  to  $-30^\circ\text{C}$  is

$$\Delta H = 313,614 - 79,457 = 234,157 \text{ J/kg}$$

### 5.3.2 Enthalpy Changes During the Freezing of Foods Calculated from Molality of Liquid Water Fraction of the Food

When pure water is frozen, a phase change from liquid water to ice occurs at the freezing point. The cooling curve for water, a plot of temperature against time, will show the temperature sloping toward the freezing point followed by a constant temperature at the freezing point if the rate of cooling is slow enough to permit ice crystals adequate time to develop. The constant temperature is due to the release of energy associated with the phase change; all energy removed from the system at the freezing point serves as an energy sink for the latent heat of fusion. There will be no sensible heat loss (no temperature reduction) until all water has been converted to ice.

In foods, water exists as a solution. All water-soluble compounds in the food contribute to depress the freezing point. Because water crystals consist of pure water, transformation of water to ice is accompanied by an increase in solute concentration in the unfrozen water. Because solutes lower the freezing point, ice formation in foods occurs over a range of temperature. The freezing point is only the start of ice crystal formation. A typical freezing curve of temperature versus time exhibits a section where temperature is almost constant or drops much slower than the rest of the cooling curve. This

section of the freezing curve is often referred to as the “ice crystal zone.” Generally, a high initial moisture and low heat transfer rate extends the time temperature stays in the ice crystal zone. Food temperature will drop below the freezing point even when liquid water is still present.

### 5.3.3 Freezing Point Depression by Solutes

When the solution is ideal or when the solute concentration is low, the freezing point depression is

$$\Delta t_f = K_f M$$

where  $\Delta T_f$  is the freezing point depression relative to the freezing point of water,  $0^\circ\text{C}$ ;  $K_f$  = cryoscopic constant = 1.86 for water.  $M$  = molality = mole solute/(g water/1000).

Let  $n$  = gmoles solute contained in  $w$  grams water. If the freezing point of a food is known,  $n$  can be calculated from the moisture content and the freezing point depression. Otherwise, the freezing point can be calculated from concentration of solutes and the moisture content. For highly ionized solutes such as salts of sodium and potassium, multiply the actual moles of solute by 2 to obtain  $n$ .

$$M = \frac{n(1000)}{w}$$

$n$  will be constant during the freezing process, whereas  $w$  changes as ice is formed.

Let  $T_f$  = freezing point;  $\Delta T_f = 0 - T_f = (-T_f)$

$$M = \frac{(-T_f)}{1.86}$$

### 5.3.4 Amount of Liquid Water and Ice at Temperatures Below Freezing

Basis: 1 kg food. Let  $w_o$  = original water in the mixture before freezing = weight fraction water  $\times$  1000.

$$n = \left[ \frac{(-T_f)}{1.86} \right] \left[ \frac{w_o}{1000} \right] = \frac{w_o(-T_f)}{1860}$$

At any temperature below the freezing point  $T$ ;  $\Delta T_f = 0 - T = (-T)$

$$M = \frac{(-T)}{1.86} = \frac{1000n}{w}$$

$$w = \frac{1000(n)(1.86)}{(-T)} = \frac{(1000)(w_o)(-T_f)}{1860} \left( \frac{1.86}{(-T)} \right) = \frac{w_o(-T_f)}{(-T)}$$

$w$  = the amount of liquid water in the food at temperature  $T$ . The amount of ice =  $I = w_o - w$

$$I = w_o - w_o \frac{(-T_f)}{(-T)} = w_o \left( 1 - \frac{(-T_f)}{(-T)} \right)$$

### 5.3.5 Sensible Heat of Water and Ice at Temperatures Below the Freezing Point

For an increment of temperature  $dT$ , water will lose sensible heat according to:

$$dq = w C_{pl} dT$$

The change in sensible heat for liquid water ( $q_{sl}$ ) from  $T_f$  to  $T$  is determined by:

$$q_{sl} = \int_{T_f}^T C_{pl} w_o dT = C_{pl} w_o \int_{T_f}^T \frac{(-T_f)}{(-T)} dT = C_{pl} w_o (-T_f) \ln \left[ \frac{(-T)}{(-T_f)} \right] \quad (5.22)$$

The change in sensible heat of ice ( $q_{si}$ ) from  $T_f$  to  $T$  is

$$q_{si} = \int_{T_f}^T C_{pi} w_o \left[ 1 - \frac{(-T_f)}{(-T)} \right] dT = C_{pi} w_o \left[ (T_f - T) - (-T_f) \ln \frac{(-T)}{(-T_f)} \right] \quad (5.23)$$

Because the specific heat for water or ice are in J/kg.K,  $w_o$  in the equations for  $q$  should be the mass in kilograms.

When considering the energy change associated with a temperature change from any temperature  $T_1$  below the freezing point to  $T$ , substitute the amount of liquid water at  $T_1$  for  $w_o$  and  $T_1$  for  $T_f$ .

### 5.3.6 Total Enthalpy Change

The total enthalpy change will consist of: sensible heat of fat; sensible heat of non-fat solids; sensible heat of ice; sensible heat of liquid water; and the latent heat of fusion of ice.

The total enthalpy change from the freezing point to any temperature  $T$  is

$$q = \Delta H = F C_{pf}(T_f - T) + SNF C_{psnf}(T_f - T) + q_{sw} + q_{si} + I(334,860)$$

Values for the specific heats of water, ice, fat and non-fat solids were given earlier in this chapter.

**Example 5.7.** Boneless broiler breast meat contains 70.6% water, 24.0% protein, 1.2% ash, and 4.2% fat. The freezing point is  $-1.2^\circ\text{C}$ . If this meat is marinated by adding salt solution to obtain a weight gain over the unmarinated meat of 15% and a net salt (NaCl) content of 1.0%, calculate (a) the new freezing point and (b) the enthalpy change as the marinated meat is frozen to  $-18^\circ\text{C}$  from the new freezing point per kg of marinated meat.

Basis: 1 kg original meat

$$M = 1.2/1.86 = 0.645; \quad g \text{ water} = 706$$

$$n = M(w_o)/1000 = .645(706)/1000 = 0.455 \text{ gmoles}$$

For marinated meat:

$$\text{mass} = 1 + 0.15(1) = 1.15 \text{ kg}$$

$$\text{Salt} = \text{NaCl} = 0.01(1.15) = 0.015 \text{ kg}$$

$$\text{Water} = 1.15 - 0.015 - (1 - 0.706) = 0.841 \text{ kg} = 841 \text{ g}$$

Consider NaCl to have two moles solute/mole NaCl; the  $\text{Na}^+$  and  $\text{Cl}^-$  component.

The molecular weight of NaCl = 35.5.

$$\text{Total gmoles solute in the marinated meat} = 2(0.015)(1000)/35.5 + 0.455 = 1.301.$$

The molality of solutes in marinated meat:

$$M = 1.301(1000)/841 = 1.547$$

$$(a) T_f = 0 - 1.547(1.86) = -2.9^\circ\text{C}$$

$$(b) \text{ At } -18^\circ\text{C:}$$

$$w_o = 841 = 0.841 \text{ kg}$$

$$w = 841(2.9/18) = 135.5 \text{ g} = 0.1355 \text{ kg}$$

$$I = 841 - 135.5 = 705.5 \text{ g} = 0.1355 \text{ kg}$$

$$q_{sl} = 4186.8 (0.841) (2.9) \ln (18/2.9) = 18,642 \text{ J}$$

$$q_{si} = 2093.4 (.841) [ \{-2.9 - (-18)\} - 2.9 \ln (18/2.9)] = 17,263 \text{ J}$$

$$\text{Total } q = 18,642 + 17,263 + \text{SNF } (837.36)(18 - 2.9) + F(1674.72)(18 - 2.9) + I (334.860)$$

Because the original fat content was 4.2% and there was no fat addition during marination,  $F = 0.042 \text{ kg}$ . The solids non-fat will consist of protein (24%), ash (1.2%), and added salt (0.015 kg) .  $\text{SNF} = 0.24 + 0.012 + 0.015 = 0.267 \text{ kg}$ .

$$\begin{aligned} \text{Total } q &= 18,642 + 17,263 + .267(837.36)(18 - 2.9) \\ &\quad + .042(1674.72)(18 - 2.9) + 0.1355(334.860) \\ &= 84,910 \text{ J} \end{aligned}$$

Although the original basis is 1 kg of unmarinated meat, note that the amount of water, ice, fat, and solids non-fat are those present in the marinated product, therefore the calculated  $q$  will be the enthalpy change per kg of marinated meat.

### 5.3.7 Specific Heats of Gases and Vapors

From Chapter 4, the specific heat of gases depends upon whether the process is carried out at constant pressure or at constant volume. If a gas is heated by blowing air across heating elements, the process is a constant pressure process. The specific heat is designated by  $C_p$ , the specific heat at constant pressure. The heat required to raise the temperature of a gas with mass,  $m$ , at constant pressure equal the change in enthalpy,  $\Delta H$ . The enthalpy change associated with a change in temperature from a reference temperature  $T_o$  to  $T_2$  is:

$$\Delta H = m \int_{T_o}^{T_2} C_p dT = mC_{pm}(T_2 - T_o) \quad (5.24)$$

$C_{pm}$  is the mean specific heat over the temperature range  $T_o$  to  $T_2$ , and  $C_p$  is the expression for specific heat as a function of  $T$ .

If a gas is heated from any temperature  $T_1$  to a final temperature  $T_2$ , the change in enthalpy accompanying the process must be calculated as follows:

$$\Delta H = q = mC_{pm}(T_2 - T_o) - mC'_{pm}(T_1 - T_o) \quad (5.25)$$

where  $C'_{pm}$  is mean specific heat from the reference temperature  $T_o$  to  $T_1$ . Tabulated values for the mean specific heat of gases are based on ambient temperature of  $77^\circ\text{F}$  or  $25^\circ\text{C}$ , as the reference temperature. Table 5.2 lists mean specific heats of common gases in the American Engineering System of units, and Table 5.3 lists specific heats of the same gases in SI. Values for  $C_{pm}$  in Tables 5.2 and 5.3 change very little at temperatures ordinarily used in food processes, therefore,  $C_{pm}$  based on the temperature range from  $T_o$  to  $T_2$  can be used for  $\Delta H$  between  $T_1$  and  $T_2$  with very little error in comparison with the use of Equation (5.25).

**Table 5.2** Mean Heat Capacities of Various Gases from 77°F to Indicated Temperature.

Temperature (°F)	Mean specific heat in BTU/lb(°F)				
	O <sub>2</sub>	N <sub>2</sub>	CO <sub>2</sub>	H <sub>2</sub> O vapor	Air
77	0.219	0.248	0.202	0.446	0.241
100	0.219	0.248	0.203	0.446	0.241
200	0.221	0.249	0.209	0.448	0.241
300	0.223	0.249	0.216	0.451	0.242
400	0.224	0.250	0.221	0.454	0.242
500	0.226	0.250	0.226	0.457	0.243
600	0.228	0.251	0.231	0.461	0.245
700	0.230	0.253	0.236	0.456	0.246

Source: Calculated from Mean molal heat capacity of gases at constant pressure. In Harper, J. C. 1976. *Elements of Food Engineering*. AVI Publishing Co., Westport, Conn.

**Example 5.8.** Calculate the heating requirement for an air drier that uses 2000 ft<sup>3</sup>/min of air at 1 atm and 170°F, if ambient air at 70°F is heated to 170°F for use in the process.

**Solution:**

$$q = mC_{pm}(170 - 70)$$

m, the mass of air/min going through the drier, in pounds, will be calculated using the ideal gas

**Table 5.3** Mean Heat Capacities of Various Gases from 25°C to Indicated Temperature.

Temperature (°C)	Mean specific heat J/kgK				
	O <sub>2</sub>	N <sub>2</sub>	CO <sub>2</sub>	H <sub>2</sub> O vapor	Air
25	916	1037	847	1863	1007
50	919	1039	858	1868	1008
100	926	1041	879	1880	1011
125	930	1043	889	1886	1012
150	934	1044	899	1891	1014
175	936	1045	910	1897	1015
200	938	1047	920	1903	1017
250	945	1049	941	1914	1020
300	952	1052	962	1925	1023
350	960	1054	983	1937	1026

Source: Calculated from Mean molal heat capacity of gases at constant pressure. In Harper, J.C. 1976 *Elements of Food Engineering*. AVI Publishing Co., Westport, Conn.

equation.

$$PV = \frac{m}{M}RT; \quad M = \text{mol.wt.air} = 29 \text{ lb/lb mole} \\ = 126.14 \text{ lb/min}$$

From Table 5.2,  $C_{pm}$  is constant at  $0.241 \text{ BTU}/(\text{lb} \cong ^\circ\text{F})$  up to  $200^\circ\text{F}$ , therefore it will be used for the temperature range  $70^\circ\text{F}$  to  $170^\circ\text{F}$ .

$$= 126.13 \text{ lb/min} \\ m = \frac{PVM}{RT} = \frac{14.7(144) \text{ lb/ft}^2 [2000 \text{ ft}^3/\text{min}] [29 \text{ lb/lbmole}]}{[1545 \text{ ft lb}_f/\text{lbmole } ^\circ\text{R}] (460 + 170) ^\circ\text{R}} \\ q = 126.13 \frac{\text{lb}}{\text{min}} \times \frac{0.241 \text{ BTU}(170 - 70) ^\circ\text{F}}{(\text{lb } ^\circ\text{F})} = 3039.8 \text{ BTU/min}$$

**Example 5.9.** How much heat would be required to raise the temperature of  $10 \text{ m}^3/\text{s}$  of air at  $50^\circ\text{C}$  to  $120^\circ\text{C}$  at 1 atm?

**Solution:**

Using Equation (5.25):

$$C'_{pm} \text{ at } T_1 = 50^\circ\text{C} = 1008 \text{ J}/(\text{kg} \cong \text{K}) \\ C'_{pm} \text{ at } T_2 = 120^\circ\text{C} = 1012 \text{ J}/(\text{kg} \cong \text{K})$$

The mean specific heat from  $T_0$  to  $50^\circ\text{C}$  and from  $T_0$  to  $120^\circ\text{C}$  are different enough to require the use of Equation (5.25) to determine  $\Delta H$ . The reference temperature,  $T_0$ , for  $C_{pm}$  in Table 5.3 is  $25^\circ\text{C}$ . The mass is determined using the ideal gas equation.

$$m = \frac{PVM}{RT} = \frac{1 \text{ atm}(10 \text{ m}^3)/\text{s} [29 \text{ kg/kg mole}]}{0.08206 [\text{m}^3 \text{ atm}/\text{kg mole K}] (50 + 273) (\text{K})} = 10.94 \text{ kg/s} \\ q = 10.94(1012)(120 - 25) - 10.94(1008)(50 - 25) = 776.08 \text{ kJ/s} = 776.08 \text{ kW}$$

## 5.4 PROPERTIES OF SATURATED AND SUPERHEATED STEAM

Steam and water are the two most used heat transfer mediums in food processing. Water is also a major component of food products. The steam tables that list the properties of steam are a very useful reference when determining heat exchange involving a food product and steam or water. At temperatures above the freezing point, water can exist in either of the following forms.

**Saturated Liquid:** Liquid water in equilibrium with its vapor. The total pressure above the liquid must be equal to or be higher than the vapor pressure. If the total pressure above the liquid exceeds the vapor pressure, some other gas is present in the atmosphere above the liquid. If the total pressure above a liquid equals the vapor pressure, the liquid is at the boiling point.

**Saturated Vapor:** This is also known as saturated steam and is vapor at the boiling temperature of the liquid. Lowering the temperature of saturated steam at constant pressure by a small increment

will cause vapor to condense to liquid. The phase change is accompanied by a release of heat. If heat is removed from the system, temperature and pressure will remain constant until all vapor is converted to liquid. Adding heat to the system will change either temperature or pressure or both.

**Vapor-Liquid Mixtures:** Steam with less than 100% quality. Temperature and pressure correspond to the boiling point; therefore, water could exist either as saturated liquid or saturated vapor. Addition of heat will not change temperature and pressure until all saturated liquid is converted to vapor. Removing heat from the system will also not change temperature and pressure until all vapor is converted to liquid.

**Steam Quality:** The percentage of a vapor-liquid mixture that is in the form of saturated vapor.

**Superheated Steam:** Water vapor at a temperature higher than the boiling point. The number of degrees the temperature exceeds the boiling temperature is the *degrees superheat*. Addition of heat to superheated steam could increase the superheat at constant pressure or change both the pressure and temperature at constant volume. Removing heat will allow the temperature to drop to the boiling temperature where the temperature will remain constant until all the vapor has condensed.

### 5.4.1 The Steam Tables

The steam tables are tabulated values for the properties of saturated and superheated steam.

#### 5.4.1.1 The Saturated Steam Table

The saturated steam table consists of entries under the headings of temperature, absolute pressure, specific volume, and enthalpy. A saturated steam table is in the Appendix (Tables A.3 and A.4).

The temperature and absolute pressure correspond to the boiling point, or the temperature and pressure under which steam can be saturated. The absolute pressure at a given temperature is also the vapor pressure.

The rest of the table consists of three general headings each of which are subdivided into saturated liquid, evaporation, and saturated vapor. The entries under saturated liquid give the properties of liquid water at the indicated temperature. The entries under saturated vapor give the properties of steam at the boiling point. The entries under evaporation are changes due to the phase transformation and are the difference between the properties of saturated vapor and saturated liquid.

Specific volume is the reciprocal of the density. It is the volume in cubic feet occupied by 1 lb of water or steam under the conditions given.

Enthalpy is the heat content of a unit mass of steam or water at the indicated temperature and pressure. Enthalpy values in the steam tables are calculated from a base temperature of 0°C. The energy change associated with a change in temperature or pressure of steam is the difference in the initial and final enthalpies. The following examples illustrate the use of the steam tables. In these examples, atmospheric pressure is specified and may not be a standard atmosphere. Please refer to the section on “Absolute Pressure and Temperature” in Chapter 4 for a discussion of the difference between atmospheric pressure and a standard atmosphere.

**Example 5.10.** At what vacuum would water boil at 80°F? Express this in (a) inches of mercury vacuum (given: atm pressure = 30 in. Hg), (b) absolute pressure in pascals.



**Solution:**

From steam tables, Appendix Table A.3: The boiling pressure for water at 80°F is 0.50683 psia.

(a) Inches Hg absolute pressure

$$= 0.5068 \frac{\text{lb}}{\text{in.}^2} \left[ \frac{2.035 \text{ in. Hg}}{\text{lb/in.}^2} \right] = 1.03 \text{ in. Hg absolute}$$

$$\text{Vacuum} = 30 \text{ in. Hg} - 1.03 \text{ in. Hg} = 28.97 \text{ in. Hg vacuum}$$

(b) Pressure

$$= \frac{0.50683 \text{ lb}_f}{\text{in.}^2} \left[ \frac{6894.757 \text{ Pa}}{\text{lb/in.}^2} \right] \\ = 3.494 \text{ kPa absolute pressure}$$

**Example 5.11.** If 1 lb of water at 100 psig and 252°F is allowed to expand to 14.7 psia, calculate (a) the resulting temperature after expansion and (b) the quantity of vapor produced.

**Solution:**

The absolute pressure = 100 + 14.7 = 114.7 psia. At 252°F, water will not boil until the pressure is reduced to 30.9 psia. The water therefore is at a temperature much below the boiling point at 114.7 psia and it would have the properties of liquid water at 252°F.

- (a) After expansion to 14.7 psia, the boiling point at 14.7 psia is 212°F. Part of the water will flash to water vapor at 212°F and the remaining liquid will also be at 212°F.  
 (b) The enthalpy of water at 252°F is ( $h_f$  at 252°F) 220.62 BTU/lb. Basis: 1 lb H<sub>2</sub>O. Heat content = 220.62 BTU. When pressure is reduced to 14.7 psia, some vapor will be formed, but the total heat content of both vapor and liquid at 212°F and 14.7 psia will still be 220.62 BTU.

If  $x$  = wt vapor produced,  $1 - x$  = wt water at 212°F and 14.7 psia:

$$x(h_g) + (1 - x)(h_f) = 220.62$$

$$h_g = 1150.5 \text{ BTU/lb}; h_f = 180.17 \text{ BTU/lb}$$

$$x(1150.5) + (1 - x)(180.17) = 220.62$$

$$x = \frac{220.62 - 180.17}{1150.5 - 180.17} = \frac{40.45}{970.33} = 0.0417 \text{ lb H}_2\text{O}$$

**Example 5.12.** If water at 70°F is introduced into an evacuated vessel, initially at 0 psia, what would be the pressure inside the vessel when equilibrium is finally attained at 70°F?

**Solution:**

Because the vessel is completely evacuated, the gaseous phase after introduction of water will be 100% water vapor. Upon introduction, water will vaporize until the pressure of water in the space above the liquid equals the vapor pressure.

$$\text{Pressure} = \text{vapor pressure of water at } 70^\circ\text{F}$$

From steam tables, pressure at 70°F = 0.36292 psia.

**Example 5.13.** If the vessel in Example 5.11 had initially 14.7 psia absolute pressure and contained completely dry air, what would be the absolute pressure after introducing the water, assuming that none of the original air had escaped during the process?

**Solution:**

$$\begin{aligned}\text{Pressure} &= \text{partial pressure of air} + \text{partial pressure of water (volume is constant)} \\ &= \text{original pressure of dry air} + \text{vapor pressure of water} \\ &= 14.7 + 0.36292 = 15.063 \text{ psia}\end{aligned}$$

**Example 5.14.** How much heat would be given off by cooling steam at 252°F and 30.883 psia to 248°F, at the same pressure?

**Solution:**

First, check the state of water at 30.883 psia and 252°F and 248°F. From steam tables, the boiling point of water at 30.883 psia is 252°F. Therefore, steam at 252°F and 30.883 psia is saturated vapor. At 30.883 psia and 248°F, water will be in the liquid state, because 248°F is below the boiling temperature at 30.883 psia.

$$\text{Heat given off} = q = h_g \text{ at } 252^\circ\text{F} - h_f \text{ at } 248^\circ\text{F}$$

From steam tables, Appendix Table A.4

$$\begin{aligned}h_g \text{ at } 252^\circ\text{F} &= 1164.78 \text{ BTU/lb} \\ h_f \text{ at } 248^\circ\text{F} &= 216.56 \text{ BTU/lb} \\ q &= 1164.78 - 216.56 = 948.22 \text{ BTU/lb}\end{aligned}$$

Saturated steam is a very efficient heat transfer medium. Note that for only a 2°F change in temperature, 948 BTU/lb of steam is given off. The heat content of saturated vapors come primarily from the latent heat of vaporization, and it is possible to extract this heat simply by causing a phase change at constant temperature and pressure.

#### 5.4.1.2 The Superheated Steam Tables

A superheated steam table is in Appendix Table A.2. Both temperature and absolute pressure must be specified to accurately define the degree of superheat. From the temperature and absolute pressure, the specific volume  $v$  in  $\text{ft}^3/\text{lb}$  and the enthalpy  $h$  in BTU/lb can be read from the table. Example problems on the use of the superheated steam tables are as follows.

**Example 5.15.** How much heat is required to convert 1 lb of water at 70°F to steam at 14.696 psia and 250°F?

**Solution:**

First determine the state of steam at 14.696 psia and 250°F. At 14.696 psia, the boiling point is 212°F. Steam at 250°F and 14.696 psia is superheated steam. From the superheated steam table,  $h$  at

250°F is 1168.8 BTU/lb.

$$\begin{aligned}\text{Heat required} &= h_g \text{ at } 250^\circ\text{F and } 14.696 \text{ psia} - h_f \text{ at } 70^\circ\text{F} \\ &= 1168.8 \text{ BTU/lb} - 38.05 \text{ BTU/lb} \\ &= 1130.75 \text{ BTU/lb}\end{aligned}$$

**Example 5.16.** How much heat would be given off by cooling superheated steam at 14.696 psia and 500°F to 250°F at the same pressure?

**Solution:**

Basis: 1 lb of steam.

$$\begin{aligned}\text{Heat given off} &= q = h \text{ at } 14.696 \text{ psia and } 500^\circ\text{F} - h_g \text{ at } 14.696 \text{ psia and } 250^\circ\text{F} \\ &= 1287.4 - 1168.8 = 118.6 \text{ BTU/lb}\end{aligned}$$

Superheated steam is not a very efficient heating medium. Note that a 250°F change in temperature is accompanied by the extraction of only 118.6 BTUs of heat.

### 5.4.1.3 Double Interpolation from Superheated Steam Tables

Because the entries in the table are not close enough to cover all conditions, it may be necessary to interpolate between entries to obtain the properties under a given set of conditions. In the case of superheated steam where both temperature and pressure are necessary to define the state of the system, a double interpolation is sometimes necessary. The following example shows how the double interpolation is carried out.

**Example 5.17.** Calculate the enthalpy of superheated steam at 320°F and 17 psia.

**Solution:**

Entries in Appendix Table A.2 show enthalpies at 15 and 20 psia and 300°F and 350°F. The tabular entries and the need for interpolation are as follows:

Enthalpy		
P (psia)	300°F	350°F
15	1192.5	1216.2
17	?	?
20	1191.4	1215.4

Enthalpies at 300°F and 350°F at 17 psia are obtained by interpolating between 15 and 20 psia at each temperature.

At 300°F: P = 15, h = 1192.5; P = 20, h = 1191.4; at P = 17, h = ?

$$h_{(300^\circ\text{F} \wedge 17 \text{ psia})} = 1191.4 + \frac{(1192.5 - 1191.4)}{(20 - 15)}(20 - 17) = 1192.06 \text{ BTU/lb}$$

At 350°F: P = 15, h = 1216.2; P = 20, h = 1215.4; at P = 17, h = ?

$$h_{(350^\circ\text{F} \wedge 17 \text{ psia})} = 1215.4 + \frac{(1216.2 - 1215.4)}{(20 - 15)}(20 - 17) = 1215.88 \text{ BTU/lb}$$

Now it is possible to interpolate between 300°F and 350°F to obtain the enthalpy at 17 psia and 320°F.

$$h = 1215.88 - \frac{(1215.88 - 1192.06)}{(350 - 300)}(350 - 320) = 1215.88 - 14.29 = 1201.59 \text{ BTU/lb}$$

#### 5.4.2 Properties of Steam Having Less Than 100% Quality

If steam is not 100% vapor, the properties can be determined on the basis of the individual properties of the component.

If  $x = \% \text{ quality}$

$$v = xv_g + (1 - x)v_f$$

$$h = xh_g + (1 - x)h_f$$

**Example 5.18.** Calculate the enthalpy of steam at 252°F having 80% quality.

**Solution:**

From the saturated steam tables, Appendix Table A.3: At 252°F, saturated steam or water has the following properties:

$$h_f = 220.62 \text{ BTU/lb}$$

$$h_g = 1164.78 \text{ BTU/lb}$$

$$h = 1164.78(0.8) + 220.62(0.2) = 931.82 + 44.12 = 975.94 \text{ BTU/lb}$$

Note that only the temperature of steam is given in this problem. If either the temperature or pressure is given, but not both, steam is at the boiling point.

### 5.5 HEAT BALANCES

Heat balance calculations are treated in the same manner as material balances. The amount of heat entering a system must equal the amount of heat leaving a system, or:

$$\text{Heat in} = \text{heat out} + \text{accumulation}$$

At a steady state, the accumulation term is zero and heat entering the system must equal what leaves the system.

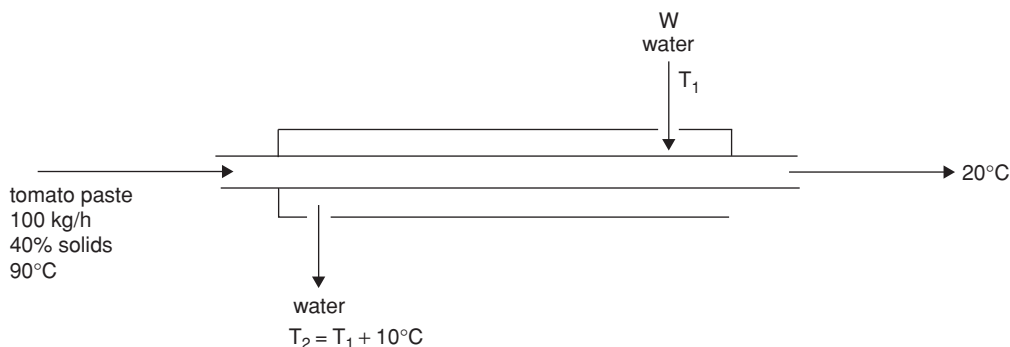
Heat balance problems are facilitated by using diagrams that show process streams bringing heat and taking heat out of a system.

**Example 5.19.** Calculate the amount of water that must be supplied to a heat exchanger that cools 100 kg/h of tomato paste from 90°C to 20°C. The tomato paste contains 40% solids. The increase in water temperature should not exceed 10°C while passing through the heat exchanger. There is no mixing of water and tomato paste in the heat exchanger.

**Solution:**

The diagram of the system is shown in Fig. 5.3.

This problem may be solved by assuming a datum from which enthalpy calculations are made. This datum temperature is the lowest of the process stream temperatures. Let  $T_1 = \text{inlet water}$



**Figure 5.3** Diagram of heat exchange during cooling of tomato paste in Example 5.19.

temperature = 20°C, and  $T_2$  = exit water temperature = 20 + 10 = 30°C. Let 20°C be the reference temperature for enthalpy calculations. The specific heat of water = 4187 J/(kg  $\cong$  K), and that of tomato paste is obtained, using Equation (5.7).

$$C_{\text{avg}} = 3349(0.6) + 837.36 = 2846.76 \text{ J/(kg } \cong \text{ K)}$$

Heat content of entering tomato paste:

$$q_1 = (100 \text{ kg}) [2846.76 \text{ J/(kg } \cong \text{ K})](90 - 20) \text{ K} = 19.927 \text{ MJ}$$

Heat content of tomato paste leaving system:

$$q_2 = 100 \text{ kg} [2846.76 \text{ J/(kg } \cong \text{ K})](20 - 20) \text{ K} = 0$$

Let  $W$  = kg water entering the system

$$q_3 = W \text{ kg} [4187 \text{ J/(kg } \cong \text{ K})](20 - 20) \text{ K} = 0$$

$$q_4 = \text{heat content of water leaving the system}$$

$$= W \text{ kg} [4187 \text{ J/(kg } \cong \text{ K})](30 - 20) \text{ K} = 41,870(W) \text{ J}$$

The heat balance is

$$q_1 + q_3 = q_2 + q_4$$

Because  $q_2$  and  $q_3 = 0$ ,  $q_1 = q_4$ , and:

$$(19.927 \text{ MJ}) \frac{10^6 \text{ J}}{\text{MJ}} = 41,871(W) \text{ J}$$

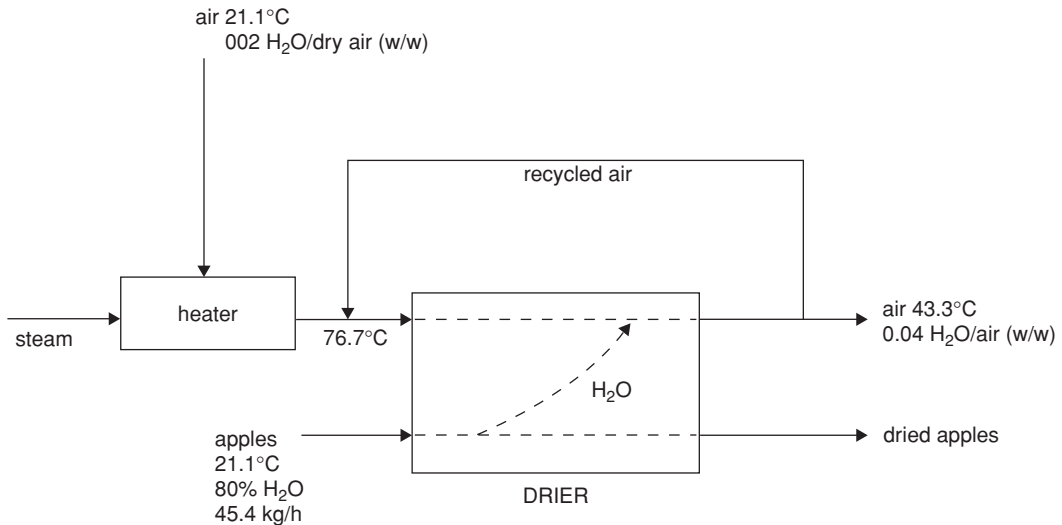
$$W = \frac{19.927 \times 10^6}{41870} = 475.9 \text{ kg}$$

The heat balance may also be expressed as follows:

Heat gain by water = heat loss by the tomato paste

$$100 \text{ kg} \left( 2846.76 \frac{\text{J}}{\text{kg} \cdot \text{K}} \right) (90 - 20) \text{ K} = W \left( 4187 \frac{\text{J}}{\text{kg} \cdot \text{K}} \right) \cdot (T_1 + 10 - T_1) \text{ K}$$

$$41870W = 100(2846.76)(70); \quad W = 475.9 \text{ kg}$$



**Figure 5.4** Diagram of material and energy balance in apple dehydration.

If there is no mixing and exchange of mass between process streams, calculation is simplified by equating heat gain by one process stream to heat loss by the other. However, when mixing and material transfer occurs, a heat balance based on heat content of each stream entering and leaving the system will simplify analysis of the problem.

**Example 5.20.** Calculate the amount of saturated steam at 121.1°C that must be supplied to a dehydrator per hour. Steam condenses in the heater, which heats the drying air from steam to water at 121.1°C. The dehydrator is operated as follows: Apples at 21.1°C enter the dehydrator with 80% moisture and leave the dehydrator at 37.7°C and 10% moisture. One hundred pounds per hour of fresh apples enter the drier. Fresh air at 21.1°C and a humidity of 0.002 kg H<sub>2</sub>O/kg dry air enter the drier, mixes with recycled hot air until the humidity is 0.026 kg H<sub>2</sub>O/kg dry air, and is heated to 76.7°C using steam in a finned heat exchanger. Hot air leaves the drier at 43.3°C and a humidity of 0.04 kg H<sub>2</sub>O/kg dry air.

### Solution:

The diagram for the process is shown in Fig. 5.4. Use as a basis 1 h of operation.

The system looks complicated with the hot air recycling, but if the boundary of the system is enlarged as shown in Fig. 5.4, solving the problem is considerably simplified.

The problem will be solved by separating air into dry air and water vapor, apples into dry matter and water, and heat contents of each component entering and leaving the system will be calculated. The lowest temperature, 21.1°C may be used as the base temperature for heat content calculations as in the preceeding example, but because the steam tables will be used to determine the enthalpy of water and steam, it will be more consistent to use the reference temperature for the steam tables of 0°C.

The following enthalpies are to be calculated:

Heat input =  $q$

$q_1$  = enthalpy of  $H_2O$  in entering air (vapor at  $21.1^\circ C$ )

$q_2$  = enthalpy of dry air entering at  $21.1^\circ C$

$q_3$  = enthalpy of  $H_2O$  in apples entering (liquid at  $21.1^\circ C$ )

$q_4$  = enthalpy of dry matter in apples entering at  $21.1^\circ C$

$q_5$  = enthalpy of  $H_2O$  in exit air (vapor at  $43.3^\circ C$ )

$q_6$  = enthalpy of dry air leaving at  $43.3^\circ C$

$q_7$  = enthalpy of  $H_2O$  in apples leaving (liquid at  $37.7^\circ C$ )

$q_8$  = enthalpy of dry matter in apples leaving at  $37.3^\circ C$

The heat balance is

$$q + q_1 + q_2 + q_3 + q_4 = q_5 + q_6 + q_7 + q_8$$

$q_2$  and  $q_6$  are calculated using the quantity of dry air that enters the system per hour. The latter is obtained by performing a material balance on water.

Amount of water lost by apples = amount of water gained by air

Solids balance for apples:

$$x = \frac{45.4(0.2)}{0.9} = 10.09 \text{ kg/h}$$

$$45.4(0.2) = x(0.9); \quad x = \text{wt dried apples}$$

$$\frac{\text{Water gained by air}}{\text{kg dry air}} = (0.04 - 0.002) = 0.038$$

$$\text{Water lost by apples} = 45.4 - 10.09 = 35.31 \text{ kg/h}$$

Let  $w$  = mass of dry air:

$$\text{Total water gained by air} = 0.038 w$$

$$\text{Material balance on water: } 0.038 w = 35.31$$

$$w = 929.21 \text{ kg dry air/h}$$

The mean specific heat of air is obtained from Table 5.3. These values are

$$25^\circ C: C_{pm} = 1008 \text{ J/(kg } \cong K)$$

$$50^\circ C: C_{pm} = 1007 \text{ J/(kg } \cong K)$$

These mean specific heat will be used for determining enthalpies of air at  $21.1^\circ C$  and  $43.3^\circ C$ .

From the section "Specific Heat of Solids and Liquids";

$$C_p \text{ for solids non-fat in apples} = 837.36 \text{ J/(kg } \cong K)$$

Use steam tables for heat contents of water and steam. Calculating the enthalpies:

$$q = m C_{pm}(T - 0) \text{ because } 0^\circ C \text{ is used as the reference temperature.}$$

$$q_1 = (929.21 \text{ kg dry air}) \frac{0.002 \text{ kg } H_2O}{\text{kg dry air}} (h_g \text{ at } 21.1^\circ C)$$



From Appendix Table A.4,  $h_g$  at  $21.1^\circ\text{C}$  by interpolation =  $2.54017\text{ MJ/kg}$ .

$$q_1 = 929.21(0.002)(2.54017 \times 10^6) = 4.7207\text{ MJ}$$

$$q_2 = (929.21)\text{ kg dry air}(1008)\frac{\text{J}}{\text{kg}} \cdot \text{K}(21.1 - 0) = 19.7632\text{ MJ}$$

$$q_3 = 45.4(0.8)(h_f \text{ at } 21.1^\circ\text{C})$$

From Appendix Table A.4,  $h_f$  at  $21.1^\circ\text{C}$  =  $0.08999\text{ MJ/kg}$

$$q_3 = 45.4(0.8)(0.08999 \times 10^6) = 3.2684\text{ MJ}$$

$$q_4 = (45.4)(0.2)\text{ kg dry solids} \left( \frac{837.36\text{ J}}{\text{kg} \cdot \text{K}} \right) (21.1 - 0)^\circ\text{C} = 0.16043\text{ MJ}$$

$$q_5 = (929.21\text{ kg dry air}) \left( 0.04 \frac{\text{kg H}_2\text{O}}{\text{kg dry air}} \right) (h_g \text{ at } 43.3^\circ\text{C})$$

From Appendix Table A.4,  $h_g$  at  $43.3^\circ\text{C}$  =  $2.5802\text{ MJ/kg}$

$$q_5 = (929.21)(0.04)(2.5802 \times 10^6) = 95.9019\text{ MJ}$$

$$q_6 = (929.21\text{ kg dry air}) \frac{1007\text{ J}}{\text{kg K}} (43.3 - 0)$$

$$= (929.21)(1007)(43.3) = 40.5164\text{ MJ}$$

$$q_7 = (10.09)(0.1)\text{ kg H}_2\text{O}(h_f \text{ at } 37.7^\circ\text{C})$$

From Appendix Table A.4,  $h_f$  at  $37.7^\circ\text{C}$  =  $0.15845\text{ MJ/kg}$

$$q_7 = (10.09)(0.1)(0.15845) = 0.15987\text{ MJ}$$

$$q_8 = (10.09)(0.9)\text{ kg dry matter} \left( \frac{837.36\text{ J}}{\text{kg F}} \right) (37.7 - 0)$$

$$= (10.09)(0.9)(837.36)(37.7) = 0.28667\text{ MJ}$$

The heat balance with  $q$  in MJ is

$$q + [4.7207 + 19.7632 + 3.2684 + 0.16043]$$

$$= [95.9019 + 40.5164 + 0.15988 + 0.28667]$$

$$q = 136.8648 - 27.91273 = 108.952\text{ MJ}$$

Because the basis is 1 h of operation, the heating requirement for the process is  $108.952\text{ MJ/h}$ .

From Appendix Table A.4, the heat of vaporization of steam at  $121.1^\circ\text{C}$  is  $2.199144\text{ MJ/kg}$ .

$$\text{The amount of steam supplied/h} = \frac{108.952\text{ MJ/h}}{2.199144\text{ MJ/kg}} = 49.543\text{ kg/h}$$

### Alternative solution:

The problem will be solved using a balance of heat sinks and heat source in the system. Consider the system as a whole, enclosing the recycle stream within the system boundary. Entering the system are air and apples, which both gain sensible heat through the process. Also entering the system is steam, which is the only heat source. Incoming air with the water vapor may be combined, and using an average specific heat, the sensible heat gain is easily calculated. Additional moisture in the air leaving the system is accounted for by moisture evaporation from apples, which may be assumed to

occur at the temperature the apples leave the system. Thus energy gain by vaporized water is the difference between the the enthalpy of the vapor at 43.3°C and the enthalpy of liquid at 37.7°C. The sensible heat gain by apples will be assessed with the initial weight and specific heat. In equation form, the energy balance is  $m_{ma} C_{pm} (43.3 - 21.1) + m_{ap} C_p (37.7 - 21.1) + m_v (h_{g@43.3\text{ C}} - h_{f@37.7\text{ C}}) = m_s (h_{fg@121.1\text{ C}})$  where  $m_{ma}$ ,  $m_{ap}$ , and  $m_s$  are the mass of moist air, wet apples, and steam, respectively.  $C_{pm}$  is the average specific heat of the moist air, and  $C_p$  is the specific heat of the wet apples. The specific heat of apples =  $0.8(4186.8) + 0.2(837.36) = 3516.9 \text{ J/kg} \cong \text{K}$ . Mass of vapor is the same as the amount removed from apples and is determined by a material balance:

$$m_v = 45.4 - \frac{45.4(0.2)}{0.9} = 35.31 \frac{\text{kg}}{\text{h}}$$

Air will gain  $(0.04 - 0.002)$  or  $0.038 \text{ kg water/kg dry air}$ . Thus, the mass of dry air entering the system is  $35.31/0.038 = 929.21 \text{ kg}$ . The mass of moist air is

$$m_{ma} = \left[ \frac{929.21 \text{ kg dry air}}{\text{h}} \right] \left[ \frac{1.002 \text{ kg moist air}}{\text{kg dry air}} \right] = 931.07 \frac{\text{kg}}{\text{h}}$$

The specific heat of air and water, respectively at 50°C are 1008 and  $1868 \text{ J/kg} \cong \text{K}$ . Thus, the mean specific heat for moist air is  $C_{pm} = [1008 (1) + 1868 (0.002)]/(1/1.002) = 1009.7 \text{ J/kg} \cong \text{K}$ .

The enthalpy of vapor at 43.3°C ( $h_{g@43.3\text{ C}}$ ) from Appendix Table A.4 by interpolation between 42.5°C and 45°C is

$$2.5788 + [(2.5832 - 2.5788)/2.5](43.3 - 42.5) = 2.58021 \text{ MJ/kg}.$$

The enthalpy of liquid at 37.7°C =  $0.15816 \text{ MJ/kg}$ . The latent heat of evaporation at 121.1°C ( $h_{fg@121.1\text{ C}}$ ) from Appendix Table A.4 by interpolation is

$$2.20225 - [(2.20225 - 2.19519)/(122.5 - 120)](121.1 - 120) = 2.19914 \text{ J/kg}.$$

Solving for  $m_s$ :

$$\begin{aligned} m_s &= \frac{(931.07)(1009.7)(22.2) + (45.4)(3516.9)(16.6) + (35.31)(2.42236 \times 10^6)}{2.19914 \times 10^6} \\ &= 49.58 \frac{\text{kg}}{\text{h}} \end{aligned}$$

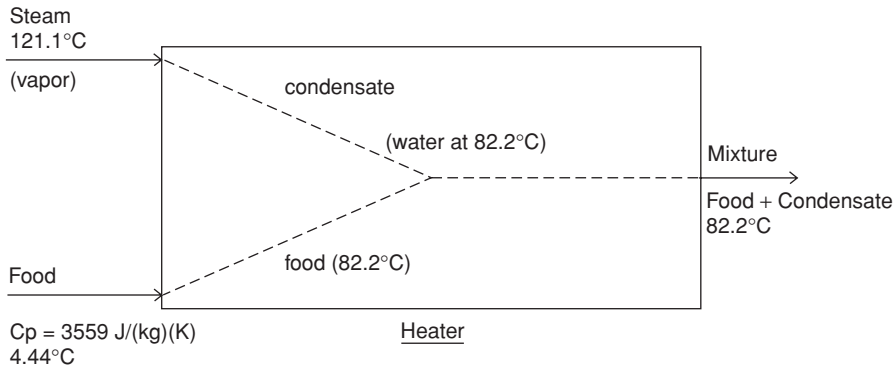
**Example 5.21.** Calculate the amount of steam at 121.1°C (250°F) that must be added to 100 kg of a food product with a specific heat of  $3559 \text{ J/(kg} \cong \text{K)}$  to heat the product from 4.44°C (40°F) to 82.2°C (180°F) by direct steam injection.

**Solution:**

The diagram of the system is shown in Fig. 5.5. Let  $x = \text{kg of steam required}$ . From steam table, the enthalpies of water and steam are as follows:

For steam at 121.1°C or 250°F,  $h_g$  from Table A.3 =  $1164.1 \text{ BTU/lb} = 2.70705 \text{ MJ/kg}$

$$\frac{\text{Heat loss steam}}{\text{kg}} = h_g \text{ at } 121.1^\circ\text{C} - h_f \text{ at } 82.2^\circ\text{C}$$



**Figure 5.5** Diagram of a direct steam injection process for heating a food product.

For water at 82.2°C or 180°F,  $h_f = 148.00 \text{ BTU/lb} = 0.34417 \text{ MJ/kg} = 2.70705 - 0.34417 = 2.36288 \text{ MJ/kg}$

Total heat loss steam =  $x(2.36288) \text{ MJ}$

Heat gain by product =  $100\text{kg}[3559 \text{ J/(kg} \cong \text{K)}](82.2 - 4.44) \text{ K} = 27.67478 \text{ MJ}$

$$x(2.36288) = 27.67478$$

$$x = 11.71 \text{ kg steam required}$$

## PROBLEMS

- 5.1. What would be the pressure generated when milk is heated to 135°C in a closed system? If the system is not pressurized, can this temperature be attained?
- 5.2. A method for heating food with saturated steam at temperatures below the boiling point of water is by carrying out the process under a vacuum. At what vacuum should a system be operated to heat a material with saturated steam at 150°F.
- 5.3. If a retort indicates a pressure of 15 psig but mercury in glass thermometer registers only 248°F, what does this indicate? Assume that both instruments are in working order and have been properly calibrated.
- 5.4. An evaporator is operated at 15 in. Hg vacuum. What would be the temperature of the product inside the evaporator? Assume product has the same boiling point as water.
- 5.5. How much heat is required to convert 1 kg water at 20°C to steam at 120°C?
- 5.6. How much heat must be removed to convert 1 lb steam at 220°F to (a) water at 220°F; (b) water at 120°F?
- 5.7. One pound of steam at 260°F contains 80% steam and 20% liquid water. How much heat will be released by this steam when it is allowed to condense to water at 200°F?
- 5.8. At what temperature would water be expected to boil at 10 in. Hg vacuum? Atm pressure = 14.696 psia.
- 5.9. How much steam at 250°F would be required to heat 10 lb of water from 70°F to 210°F in a direct steam injection heater?

- 5.10. How much heat will be required to convert steam at 14.696 psig to superheated steam at 600°F at the same pressure?
- 5.11. Ten pounds of water at a pressure of 20 psig is heated to a temperature of 250°F. If this water is allowed to empty into an open vessel at atmospheric pressure, how much of the water will remain in the liquid phase?
- 5.12. (a) If water at 70°F is introduced into an evacuated vessel where the original pressure is 0 psia, what would be the pressure inside the vessel at equilibrium? Assume no change in the temperature of water. (b) If the original pressure is 14.696 psia, what will be the final pressure?
- 5.13. Determine the heat content in BTU/lb for water (it could be liquid, saturated steam, or superheated steam) under the following conditions: (a) 180°F and 14.696 psia pressure, (b) 300°F and 14.696 psia pressure, (c) 212.01°F and 14.696 psia pressure.
- 5.14. In the formulation of a pudding mix, it is desired that the solids content of the product would be 20%. The product leaving the batch tank has a temperature of 26.67°C (80°F) and this is preheated to 90.56°C (195°F) by direct steam injection using culinary steam (saturated) at 104.4°C (220°F) followed by heating in a closed system to sterilizing temperatures. There is no further loss or gain of moisture in the rest of the process. What should be the solids content of the formulation in the batch tank such that after the direct steam injection heating the final solids content of the product will be 20%. Use Siebel's equation for calculating the specific heat of the product.
- 5.15. A fruit juice at 190°F is allowed to flash into an essence recovery system maintained at a vacuum of 29 in. of mercury. Atmospheric pressure is 29.9 in. The vapors that flash-off are rectified to produce an essence concentrate, and the juice after being stripped of the aromatic constituents is sent to an evaporator for concentration.

Assuming sufficient resident time for the juice in the system to allow equilibrium in temperature between the liquid and the vapor, calculate:

- (a) The temperature of the juice leaving the essence recovery system.
- (b) The solids content of the juice leaving the system if the original solids content is 10%. Assume no additional heat input and the latent heat for vaporization is derived from the loss in sensible heat of the liquid. The specific heat of the solids is 0.2 BTU/(lb  $\cong$  °F).
- (c) The quantity of water vaporized per 100 lb of juice entering the system.
- 5.16. An evaporator has a heat transfer surface area that would allow the transfer of heat at the rate of 100,000 BTU/h. If this evaporator is concentrating apple juice from 10% to 45% solids, under a vacuum of 25 in. Hg (atmospheric pressure is 30 in. of mercury), how much apple juice can be processed per hour?
- 5.17. Orange juice concentrate at 45% total solids leaves the evaporator at 50°C. This is frozen into slush in swept surface heat exchangers until half of the water is in the form of ice crystals prior to filling into cans, and the cans are frozen at -25°C. Assume that the sugars are all hexose sugars, and that the freezing point reduction can be determined using  $\Delta T_f = K_f m$ , where  $K_f$  = the cryoscopic constant = 1.86 and  $m$  = the molality. Calculate:
  - (a) The total heat that must be removed from the concentrate in the swept surface heat exchangers per kg of concentrate processed.
  - (b) The amount of heat that must be further removed from the concentrate in frozen storage.
  - (c) The amount of water still in the liquid phase at -25°C.

Note: The moisture content is beyond the range where Chang and Tao's correlation is applicable. Determine the freezing point by calculating the freezing point depression:  $\Delta T_b = K_f m$ . The specific heat of the solids is the same below and above freezing. The specific heat of ice is 2093.4 J/(kg·K). The heat of fusion of ice is 334860 J/kg. The juice contains 42.75% soluble solids.

- 5.18. In a falling film evaporator, fluid is pumped to the top of a column and the fluid falls down as a film along the heated wall of the column increasing in temperature as it drops. When the fluid emerges from the column, it is discharged into a vacuum chamber where the fluid drops in temperature by flash evaporation until it reaches the boiling temperature at the particular vacuum employed. If juice containing 15% solids is being concentrated to 18% solids in one pass through the heated column, and the vacuum in the receiving vessel is maintained at 25 in. Hg, calculate the temperature of the fluid as it leaves the column such that when flashing occurs the desired solids content will be obtained.
- 5.19. When sterilizing foods containing particulate solids in the Jupiter system, solids are heated separately from the fluid components by tumbling the solids in a double cone processing vessel with saturated steam contacting the solids. The fluid component of the food is heated, held until sterile, and cooled using conventional fluid heating and cooling equipment. The cooled sterile liquid is pumped into the double cone processing vessel containing the hot solids, which cools the latter and drops the pressure to atmospheric. After allowing the mixture to cool by cooling the walls of the processing vessel, the sterile mixture is filled aseptically into sterile containers
- (a) Meat and gravy sauce is being prepared. Beef cubes containing 15% solids non-fat, 22% fat, and 63% water are heated from 4°C to 135°C, during which time condensate accumulates within the processing vessel with the meat. Saturated steam at 135°C is used for heating. Calculate the total amount of meat and condensate at 135°C.
  - (b) The gravy mix is of equal weight as the raw meat processed and consists of 85% water and 15% solids non-fat. Calculate the temperature of the mixture after equilibration if the gravy mix is at 20°C when it is pumped into the processing chamber containing the meat at 135°C.
- 5.20. The chillers in a poultry processing plant cool broilers by contacting the broilers with a mixture of water and ice. Broilers enter at 38°C and leave the chillers at 4°C. USDA requires an overflow of 0.5 gallons of water per broiler processed, and this must be replaced with fresh water to maintain the liquid level in the chiller. Melted ice is part of this overflow requirement. If a plant processes 7000 broilers/h, and the broilers average 0.98 kg with a composition of 17% fat, 18% solids non-fat, and 65% water, calculate the ratio by weight of ice to fresh water that must be added into the chiller to meet the overflow requirement and the cooling load. Fresh water is at 15°C, and the overflow is at 1.5°C.
- 5.21. Saturated steam at 280°F is allowed to expand to a pressure of 14.696 psia without a loss of enthalpy. Calculate: (a) the temperature, (b) the weight of high pressure steam needed to produce 100 m<sup>3</sup>/min of low pressure steam at 14.696 psia and the temperature calculated in (a).
- 5.22. In one of the systems for ultra-high-temperature sterilization, milk enters a chamber maintained at 60 psia and 800°F in an atmosphere of superheated steam where it discharges from a plenum into vertical tubes where it falls down in a thin film while exposed to the steam. The milk will be at the boiling temperature at 60 psia on reaching the bottom of the heating chamber. After a sterilizing hold time at constant temperature, the milk is discharged into a vacuum chamber for rapid cooling. If the vacuum chamber is at 15 in. Hg vacuum, calculate:
- (a) The temperature of the milk leaving the flash chamber.
  - (b) The total solids content. Raw milk enters the heater at 2°C, and contains 89% water, 2% fat, and 9% solids non-fat. Given: The enthalpy of superheated steam at 60 psia and 800°F is 1431.3 BTU/lb. Saturated steam temperature at 60 psia is 292.7°F. The enthalpy of saturated liquid ( $h_f$ ) at 292.7°F is 260.7 BTU/lb.
- 5.23. (a) Calculate the freezing point of fresh strawberries that contain 8.5% soluble solids, 1% insoluble solids, and 90.5% water. Assume that the average molecular weight of the soluble solids is 261.

- (b) If sucrose is added to the above strawberries in the ratio 1 part of sugar to 3 parts strawberries by weight, calculate the new freezing point.
- (c) Calculate the change in enthalpy of the sugared strawberries from 20°C to −20°C.
- 5.24. A food mix containing 80% solids non-fat and 20% fat on a dry basis is to be extruded. Water is added continuously along with the product into the extruder and the mixture temperature increases to 135°C at a pressure of 600 kPa at the die entrance. Extrudate leaves the die and immediately expands to atmospheric pressure, releasing vapor as it exits the die. Assume that the temperature of the extrudate is 100°C immediately after leaving the die. It is desired that the moisture content of the extrudate will be 18% after the pressure reduction. Calculate the rate of moisture addition to the extruder if the solid feed originally contains 10% water and is fed at the rate of 30 kg/h.
- 5.25. A food product that contains 15% solids non-fat, 2% fat, and 83% water is to be pasteurized by heating to 95°C by direct steam infusion using 90% quality steam at 15 psig. After heating to 95°C, the product is held in a holding tube and cooled in a heat exchanger; therefore, there is no opportunity for removing added water from the steam condensate later in the process. Calculate the moisture content of the raw product entering the heater such that the desired moisture content will be obtained after heating and cooling.

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#### SUGGESTED READING

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